Seventy-eight charts of the Astrographic Chart of the Heavens, presented by the Royal Observatory, Greenwich; and ninety-six charts, presented by the Paris Observatory.

Two enlarged transparencies and lantern slide from negatives of the total eclipse of August 1905, taken at Alcalá Chisvert, Spain, presented by Count de la Baume Pluvinel.

The Early Eclipses of the Sun and Moon. By E. Nevill.

With the view of furnishing Mr Cowell with the further information he desires (*Monthly Notices*, vol. lxvi. p. 474), I have extracted from my memoir on the Observed Errors of Hansen's *Tables de la lune* my investigation of the results yielded by the observations of the ancient Eclipses of the Moon contained in Ptolemy's *Almagest*.

The eclipses and errors of Hansen's Tables are as follow :—

	Date.		Place.	Phase.	Error of Hansen's Tables.	Weight.	Remarks.
1	−720 March	19	Babylon	Beginning	-23^m	2	Total
2	−719 March	8	,,	Middle	−63	½	Mag. = 0·25 on S
3	−719 Sept.	1	,,	Beginning	−43	2	Mag. = 0·55 on N
4	−620 April	21	,,	Beginning	−44	3	Mag. = 0·25 on S
5	−522 July	16	,,	Beginning	−93	1	Mag. = 0·50 on N
6	−501 Nov.	19	,,	Beginning	−28	1	Mag. = 0·25 on S
7	−490 April	25	,,	Beginning	−28	1	Mag. = 0·17 on S
8	−382 Dec.	22	Athens	Beginning	−51	½	Small. Place uncertain
9	−381 June	18	Babylon	Beginning	−43	2	Partial
9a	,,		,,	End	−77	1	Duration $3^h\ 0^m$
10	−381 Dec.	12	,,	Beginning	−59	1	Total
11	−200 Sept.	22	,,	Beginning	−26	1	Partial
11a	,,		,,	End	−30	2	Duration $3^h\ 2^m$
12	−199 March	19	Alexandria	Beginning	−38	3	Total
13	−199 Sept.	11	,,	Beginning	−25	2	Total
13a	...		,,	Middle	−19	1	
14	−173 April	30	,,	Beginning	−44	1	Mag. = 0·58 on N
14a	...		,,	End	−46	1	Duration $2^h\ 43^m$
15	−143 Jan.	27	Rhodes	Beginning	−83	½	Mag. = 0·25 on S
16	+125 May	5	Alexandria	Beginning	−26	2	Mag. = 0·17 on S
17	+133 May	6	,,	Middle	−30	3	Total
18	+134 Oct.	20	,,	Middle	−13	2	Mag. = 0·83 on N
19	+136 March	5	,,	Middle	−48	2	Mag. = 0·50 on N

The data have been taken, primarily, from Professor Newcomb's *Researches on the Motion of the Moon*, but in certain cases I have interpreted the record in a somewhat different manner, on the strength of additional information, and have assumed that the Eclipse No. 8 was observed at Athens, the actual place of observation being left unspecified. This Eclipse No. 8 and Eclipse No. 15 observed at Rhodes stand in a different category to the others, seemingly having been selected from outside Ptolemy's usual authorities for some special reason, possibly because no eclipses observed at either Babylon or Alexandria were observed under the desired conditions. Neither has been used in the discussion which follows.

Writing—

(Tabular—Observed) Longitude $= \Delta L + \sin\alpha \,.\, A_s + \cos\alpha \,.\, A_c + \sin\beta \,.\, B_s + \cos\beta \,.\, B_c + (T - 1800) \,.\, 30'' \,.\, \delta l^* + (T - 1800)^2 \, 1'' \,.\, 0 \, A$

where α denotes the Moon's mean anomaly and β denotes the Moon's mean argument of latitude. Then the preceding data lead to the expressions—

1	$\Delta L = +11{\cdot}1$	$+{\cdot}88\,A$	$+{\cdot}47\,A_c$	$-{\cdot}01\,B_s$	$-{\cdot}99\,B_c$	$+4{\cdot}20\,\delta l^*$	$-10{\cdot}58\,A$
2	$= +28{\cdot}5$	$+{\cdot}21$	$+{\cdot}98$	$-{\cdot}15$	$-{\cdot}99$	$+4{\cdot}20$	$-10{\cdot}57$
3	$= +25{\cdot}3$	$+{\cdot}33$	$-{\cdot}94$	$+{\cdot}13$	$+{\cdot}99$	$+4{\cdot}20$	$-10{\cdot}57$
4	$= +19{\cdot}9$	$-{\cdot}31$	$+{\cdot}95$	$-{\cdot}18$	$+{\cdot}98$	$+4{\cdot}03$	$-9{\cdot}76$
5	$= +42{\cdot}7$	$+{\cdot}45$	$+{\cdot}89$	$+{\cdot}15$	$-{\cdot}99$	$+3{\cdot}87$	$-8{\cdot}99$
6	$= +12{\cdot}6$	$+{\cdot}06$	$+{\cdot}99$	$-{\cdot}14$	$+{\cdot}99$	$+3{\cdot}84$	$-8{\cdot}85$
7	$= +14{\cdot}7$	$+{\cdot}99$	$-{\cdot}07$	$-{\cdot}20$	$+{\cdot}98$	$+3{\cdot}82$	$-8{\cdot}75$
8	$= +28{\cdot}7$	$-{\cdot}78$	$-{\cdot}62$	$+{\cdot}20$	$-{\cdot}98$	$+3{\cdot}65$	$-7{\cdot}95$
9	$= +18{\cdot}9$	$+{\cdot}42$	$+{\cdot}91$	$-{\cdot}15$	$+{\cdot}99$	$+3{\cdot}64$	$-7{\cdot}93$
9a	$= +33{\cdot}8$	$+{\cdot}42$	$+{\cdot}91$	$-{\cdot}15$	$+{\cdot}99$	$+3{\cdot}64$	$-7{\cdot}93$
10	$= +35{\cdot}0$	$-{\cdot}01$	$-{\cdot}99$	$+{\cdot}07$	$-{\cdot}99$	$+3{\cdot}64$	$-7{\cdot}93$
11	$= +12{\cdot}4$	$-{\cdot}82$	$+{\cdot}57$	$-{\cdot}11$	$+{\cdot}99$	$+3{\cdot}34$	$-6{\cdot}66$
11a	$= +14{\cdot}3$	$-{\cdot}82$	$+{\cdot}57$	$-{\cdot}11$	$+{\cdot}99$	$+3{\cdot}34$	$-6{\cdot}66$
12	$= +20{\cdot}3$	$+{\cdot}97$	$-{\cdot}22$	$-{\cdot}02$	$-{\cdot}99$	$+3{\cdot}33$	$-6{\cdot}65$
13	$= +13{\cdot}5$	$-{\cdot}96$	$-{\cdot}29$	$+{\cdot}00$	$+{\cdot}99$	$+3{\cdot}33$	$-6{\cdot}65$
13a	$= +10{\cdot}2$	$-{\cdot}96$	$-{\cdot}29$	$+{\cdot}00$	$+{\cdot}99$	$+3{\cdot}33$	$-6{\cdot}65$
14	$= +25{\cdot}8$	$+{\cdot}33$	$-{\cdot}95$	$+{\cdot}12$	$+{\cdot}99$	$+3{\cdot}29$	$-6{\cdot}49$
14a	$= +27{\cdot}0$	$+{\cdot}33$	$-{\cdot}95$	$+{\cdot}12$	$+{\cdot}99$	$+3{\cdot}29$	$-6{\cdot}49$
15	$= +48{\cdot}9$	$+{\cdot}05$	$-{\cdot}99$	$-{\cdot}15$	$-{\cdot}99$	$+3{\cdot}24$	$-6{\cdot}27$
16	$= +13{\cdot}9$	$-{\cdot}95$	$-{\cdot}29$	$-{\cdot}17$	$+{\cdot}98$	$+2{\cdot}83$	$-4{\cdot}68$
17	$= +14{\cdot}1$	$-{\cdot}69$	$+{\cdot}72$	$+{\cdot}08$	$-{\cdot}99$	$+2{\cdot}81$	$-4{\cdot}63$
18	$= +6{\cdot}3$	$+{\cdot}86$	$+{\cdot}50$	$+{\cdot}08$	$+{\cdot}99$	$+2{\cdot}81$	$-4{\cdot}63$
19	$= +21{\cdot}7$	$+{\cdot}62$	$-{\cdot}78$	$+{\cdot}19$	$-{\cdot}98$	$+2{\cdot}80$	$-4{\cdot}62$

Let—

A denote the values obtained from Hansen's Tables, with his

tabular mean motions and secular accelerations replaced by Hansen's finally deduced values for these elements.

B denote the values obtained from Hansen's Tables after replacing his tabular values for the secular accelerations by those derived theoretically from the secular diminution of the eccentricity of the terrestrial orbit, removing his empirical Venus term of long period, and making the necessary corrections to the mean motions requisite to bring the amended Tables as far as possible into accord with the modern observations.

C denote the values obtained from Hansen's Tables after applying the corrections indicated by the discussion of the observations of the Moon made between 1650 and 1890, employing for the secular acceleration of the mean longitude the value $+7''\cdot43$, of the longitude of the node the value $6''\cdot07$, and of the longitude of the lunar perigee the value $-32''\cdot53$.

Then the preceding expressions yield for the tabular errors of the Tables as modified in B and C the values

No.	Date.	Tables B.—Obs. ΔL.	Tables B.—Obs. ΔE.N.P.D.	Tables C.—Obs. ΔL.	Tables C.—Obs. ΔE.N.P.D.
		′	′	′	′
1	−720 March 19	−38·1	...	−17·8	...
2	−719 March 8	−19·2	+1·2	+ 3·9	+1·9
3	−719 Sept. 1	−28·9	+1·8	−16·2	+1·1
4	−620 April 21	−24·5	+6·0	− 4·2	+5·6
5	−522 July 16	+ 2·6	−2·4	+21·2	−1·8
6	−501 Nov. 19	−26·3	+4·2	− 7·6	+3·6
7	−490 April 25	−26·2	+5·4	−11·8	+4·8
8	−382 Dec. 22	− 9·5 ?	...	+ 1·5 ?	...
9	−381 June 18	−10·4	+8·3	+ 5·9	+7·8
10	−381 Dec. 12	− 3·9	...	+ 5·8	...
11	−200 Sept. 22	−16·2	+2·0	− 3·3	+1·6
12	−199 March 19	−10·0	...	+ 0·5	...
13	−199 Sept. 11	−18·0	...	− 7·7	...
14	−173 April 30	− 4·1	+2·6	− 4·0	+1·8
15	−140 Jan. 27	+19·4 ?	−1·2	+27·3 ?	−0·8
16	+125 April 5	− 6·0	+2·5	− 0·9	+2·2
17	+135 May 6	− 4·9	...	+ 4·2	...
18	+134 Oct. 20	−12·8	+1·4	− 4·2	+1·7
19	+136 March 5	+ 1·5	−5·0	+ 7·5	−4·7

The Eclipse No. 8, whose place of observation is not stated, but has been assumed to be Athens, yields values fairly accordant with those from the other eclipses. The Eclipse No. 15 observed at Rhodes is quite discordant, indicating either that Zech was correct in supposing the time to be an hour in error, or else that the phase observed was the middle of the eclipse: the first supposition would

render the error in longitude B = $-14'\cdot2$ C = $-6'\cdot3$, and the second the values B = $-12'\cdot4$ C = $-4'\cdot5$; results in fair accord with those from the other eclipses. Neither eclipse has been included in the investigation of the errors in longitude, however, owing to the inherent uncertainty.

From the preceding, there are derived the following values for the mean outstanding errors of the Tables as amended :—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.
−720	ΔL = $-28'\cdot7$	= $-15'\cdot2$	3
−620	= $-24\cdot5$	= $-4\cdot2$	2
−504	= $-16\cdot6$	= $+0\cdot6$	2
−381	= $-8\cdot8$	= $+5\cdot9$	4
−192	= $-11\cdot7$	= $-1\cdot2$	10
+125	= $-6\cdot3$	= $+0\cdot9$	2
+134	= $-5\cdot3$	= $+2\cdot6$	6
Weighted mean	ΔL = $-12\cdot8$	= $-0\cdot9$	

It is obvious, therefore, that these eclipses cannot be reconciled with the theoretical value of the secular acceleration in mean longitude embodied in Tables B, or with any value less than that adopted in Tables C, or below $+7''\cdot40$.

Considering the correction $\Delta B''$ to the secular acceleration of the lunar node, the mean of the results from the observed phases 3, 9a, 11a, and 14 yield the expression—

			Tables B.—Obs.	Tables C.—Obs.
$+0\cdot70\,\Delta B''$	$-3\cdot62\delta l^*$	$+7\cdot91\,A$	$=-15\cdot22$	$=-0\cdot25$

whilst the mean of the results from the observed phases 4, 5, 6, 7, 8, 9, 11, 14a, and 16 gives

$$+1\cdot38\Delta B''+3\cdot59\delta l^*-7\cdot78A=+14\cdot20=+0\cdot03.$$

Hence

$$\Delta B''-0\cdot03\delta l^*+0\cdot13A=-0''\cdot49=+0''\cdot11.$$

The correction obtained is small.

The values of the corrections to the tabular E.N.P.D. regarding them all as of equal weight yield the values—

Epoch.		Obs.		Obs.	Weight.
−565	Tables B.	ΔE.N.P.D. = $+3'\cdot50$	Tables C.	ΔE.N.P.D. = $+3'\cdot26$	7
−172		= $+0\cdot85$		= $+0\cdot65$	4
+132		= $-0\cdot27$		= $-0\cdot37$	3

Regarding these as indicating a progressive change in E.N.P.D., they correspond to the values—

$$\text{Tables B.}\quad \Delta\text{E.N.P.D.}=+0'\cdot10-0'\cdot36\,(T-0)+0'\cdot043\,(T-0)^2$$
$$\text{Tables C.}\qquad\qquad =-0\cdot02-0\cdot32\,(T-0)+0\cdot046\,(T-0)^2$$

These values are too large to be real, corresponding to a change of some 20″ per century,—an impossible amount if assumed to extend up to the present time.

If they be regarded as due to the effect of corrections required by the adopted tabular value of the argument of latitude β, necessitated by an erroneous tabular value of the secular acceleration of the Moon's node, then the values indicated are—

		Obs.	Cal.		Obs.	Cal.
−565	Tables B. $\frac{1}{11}$ $\Delta\beta=$	−3′·84	−3′·84	Tables C. $\frac{1}{11}$ $\Delta\beta=$	−3′·23	−3′·23
−172		−1·45	−2·47	=	−1·05	−2·19
+132		−2·77	−1·70	=	−3·07	−1·51

corresponding to the corrections to the secular acceleration of the node of

$$\text{Tables B.}\ \ \Delta B = -4''{\cdot}25\ (T-1800)^2$$
$$\text{Tables C.}\ \ \Delta B = -3{\cdot}77\ (T-1800)^2$$

The discordance between the observed and calculated values is very great, so that such a correction can have no real weight. Nor can it be improved by any change in the tabular mean motion of the node.

Taking the values as they stand, they would appear to involve an inequality in the complete expression for the Moon's latitude with a coefficient of about a minute of arc and a period of from seven to eight hundred years. Such a term would undoubtedly reconcile these values of $\Delta\beta$, and render consistent the values found above for the correction to the secular acceleration of the node. But it could not be rendered consistent with the modern observations, for it would mean a change in the Moon's latitude of at least ten to twelve seconds of arc within the period 1820 and 1890; though in all probability the change would be at least two or three times as great, as it is very unlikely that the epoch of the term should be such as to render the amount of change a minimum.

Such a change of the Moon's latitude is quite inconsistent with the observations.

The Arabian Eclipses.

The Arabian eclipses quoted by Professor Newcomb in § 5 of his *Researches on the Motion of the Moon*, from Caussin's edition of "le Livre de la grande Table Hakémite," have to be taken into account to obtain further information on the points which have been discussed in the previous investigation of the Ptolemaic eclipses of the Moon.

Professor Newcomb points out that some of the observed altitudes of the Moon are impossible, and some of the observed altitudes of the clock stars are irreconcilable with each other. Mr Knobel, in *Monthly Notices*, vol. xxxix. pp. 338–340, suggests that these impossible values are due to errors in copying

the Arabian figures, which often differ so slightly that errors can easily be made. I have had no means of referring to the original authorities, but as the result of some inquiries made for me by Mr Marth, I think that the altitude of Arcturus on 925 April 11, which is given as 11°, should be read as 31°; that the altitude of Arcturus on 929 January 27, which is given as 18°, should be 33°; and that the altitude of the Moon on 983 March 1, which is given as 66°, should be 62°.

When the magnitude of the eclipses is given, it does not state whether they were estimated from the upper or lower limb, as is done by Ptolemy, so that when the eclipse is large it is uncertain whether the central line of eclipse passed north or south of the centre of the Sun. This renders the interpretation of the solar eclipses of 993 August 19 and 1004 January 24 somewhat uncertain; and though in each case the most probable path has been taken, some doubt must attach to the results.

The results of the investigation of these early eclipses observed by the Arabian astronomers are as follows:—

The Early Arabian Eclipses:—

Eclipses of the Sun observed at *Bagdad*:—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Remarks.
			m			
1	829 Nov. 29	Beginning	−45·9	0		Rejected.
1a	,,	End	−18·0	1		
2	923 Nov. 10	End	−16·7	1		
3	928 Aug. 17	End	−10·0	2		

Eclipses of the Sun observed at *Cairo*:—

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude.	Remarks.
4	977 Dec. 12	Beginning	− 4·5	1		
4a	,,	End	− 3·2	1		
5	978 June 8	Beginning	−19·7	2		
5a	,,	End	− 3·5	2		
6	979 May 28	Beginning	− 6·3	2		
7	985 July 20	Beginning	−24·3	½	0·25	
7a	,,	End	−14·7	½		
8	993 Aug. 19	Beginning	− 0·3	2	0·74	
8a	,,	End	−19·9	1		
9	1004 Jan. 24	Beginning	−11·1	1	0·92	

Eclipses of the Moon observed at *Bagdad*:—

No.	Date.	Phase.	Tabular Errors.	Weight.
1	854 Aug. 11	Beginning	− 7·1	1
2	856 June 21	Beginning	− 3·7	2
3	923 June 1	End	− 6·7	2
4	925 April 11	Beginning	− 4·5	1
4a	,,	End	+ 2·2	3

Eclipses of the Moon observed at *Bagdad—continued.*

No.	Date.	Phase.	Tabular Errors.	Weight.	Magnitude. Remarks.
			m		
5	927 Sept. 13	Beginning	+13·2	0	Doubtful. Time possibly calculated.
6	929 Jan. 27	Beginning	−17·8	1	Altitude corrected for error in copying.
7	933 Nov. 4	Beginning	− 1·8	2	

Eclipses of the Moon observed at *Cairo* :—

8	979 May 14	End	−11·3	½	Doubtful. Perhaps calculated time.
9	979 Nov. 6	Beginning	− 3·4	1	
9a	,,	End	−13·9	1	
10	980 May 2	End	− 4·8	1	
11	981 April 21	Beginning	− 2·8	1	Magnitude =0·25.
11a	,,	End	+ 1·6	1	
12	981 Oct. 15	Beginning	−14·8	1	Magnitude =0·42.
13	983 March 1	Beginning	− 4·9	1	Altitude corrected for error in copying.
13a	,,	End	−17·7	2	
14	986 Dec. 18	Beginning	−19·2	2	Magnitude =0·83.
15	1002 March 1	Beginning	− 2·5	2	Total.

From these observations are obtained the following expressions :—

Eclipses of the Sun :—

1	$\Delta L = +\ 7'\!\cdot\!6$	$+\cdot98\ A_s$	$+\cdot16\ A_c$	$-\cdot06\ B_s$	$+\cdot99\ B_c$	$+9\cdot70\ \delta l$ *	$-1\cdot56$ A
2	$=+\ 6\cdot7$	$-\cdot90$	$+\cdot42$	$-\cdot10$	$+\cdot99$	$+8\cdot76$	$-1\cdot28$
3	$=+\ 4\cdot5$	$-\cdot99$	$+\cdot14$	$+\cdot05$	$+\cdot99$	$+8\cdot72$	$-1\cdot26$
4	$=+\ 1\cdot7$	$-\cdot19$	$-\cdot98$	$-\cdot11$	$+\cdot99$	$+8\cdot42$	$-1\cdot18$
4a	$=+\ 1\cdot1$	$-\cdot19$	$-\cdot98$	$-\cdot11$	$+\cdot99$	$+8\cdot42$	$-1\cdot18$
5	$=+\ 5\cdot4$	$-\cdot26$	$+\cdot96$	$+\cdot01$	$-\cdot99$	$+8\cdot41$	$-1\cdot18$
5a	$=-\ 1\cdot9$	$-\cdot26$	$+\cdot96$	$+\cdot01$	$-\cdot99$	$+8\cdot41$	$-1\cdot18$
6	$=+\ 3\cdot8$	$-\cdot90$	$+\cdot42$	$-\cdot13$	$-\cdot99$	$+8\cdot40$	$-1\cdot17$
7	$=+10\cdot6$	$+\cdot99$	$-\cdot14$	$-\cdot09$	$+\cdot99$	$+8\cdot15$	$-1\cdot10$
7a	$=+\ 9\cdot1$	$+\cdot99$	$-\cdot14$	$-\cdot09$	$+\cdot99$	$+8\cdot15$	$-1\cdot10$
8	$=+\ 0\cdot1$	$+\cdot44$	$-\cdot90$	$-\cdot04$	$-\cdot99$	$+8\cdot06$	$-1\cdot08$
8a	$=+10\cdot2$	$+\cdot44$	$-\cdot90$	$-\cdot04$	$-\cdot99$	$+8\cdot06$	$-1\cdot08$
9	$=+\ 4\cdot4$	$-\cdot94$	$-\cdot33$	$-\cdot05$	$-\cdot99$	$+7\cdot96$	$-1\cdot05$

Eclipses of the Moon :—

1	$\Delta L = +\ 3\cdot6$	$-\cdot66$	$-\cdot76$	$-\cdot01$	$-\cdot99$	$+9\cdot45$	$-1\cdot49$
2	$=+\ 1\cdot9$	$+\cdot99$	$+\cdot00$	$+\cdot20$	$-\cdot98$	$+9\cdot43$	$-1\cdot48$
3	$=+\ 3\cdot4$	$-\cdot39$	$+\cdot92$	$+\cdot14$	$+\cdot99$	$+8\cdot77$	$-1\cdot28$
4	$=-\ 2\cdot3$	$+\cdot93$	$-\cdot38$	$-\cdot09$	$+\cdot99$	$+8\cdot75$	$-1\cdot27$
4a	$=+\ 1\cdot1$	$+\cdot93$	$-\cdot38$	$-\cdot09$	$+\cdot99$	$+8\cdot75$	$-1\cdot27$

Eclipses of the Moon—*continued*

6	= + 9·1	− ·90	− ·44	− ·10	+ ·99	+ 8·71	− 1.26
7	= + 0·9	− ·57	+ ·82	+ ·02	+ ·99	+ 8·64	− 1·25
9	= + 1·7	− ·96	− ·27	+ ·11	− ·99	+ 8·20	− 1·12
9a	= + 7·0	− ·96	− ·27	+ ·11	− ·99	+ 8·20	− 1·12
10	= + 2·4	+ ·80	+ ·60	+ ·06	+ ·99	+ 8·20	− 1·11
11	= + 1·4	+ ·07	+ ·99	+ ·02	− ·99	+ 8·19	− 1·11
11a	= − 0·8	+ ·07	+ ·99	+ ·02	− ·99	+ 8·19	− 1·11
12	= + 7·7	+ ·37	− ·92	− ·16	− ·99	+ 8·18	− 1·11
13	= + 2·5	− ·85	− ·53	− ·09	+ ·99	+ 8·17	− 1·10
13a	= + 8·9	− ·85	− ·53	− ·09	+ ·99	+ 8·17	− 1·10
14	= + 9·7	+ ·17	+ ·98	− ·10	+ ·99	+ 8·15	− 1·10
15	= + 1·3	− ·07	− ·99	+ ·06	− ·99	+ 7·98	− 1·06

From these there are derived the following values :—

			Tables B.—Obs.		Tables C.—Obs.		Weight.	
			ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.	ΔL.	ΔE.N.P.D.
Eclipses of the Sun :—								
			′	′	′	′		
1	829 Nov.	29	+ 2·8	...	+ 5·1	...	1	...
2	923 Nov.	10	+ 3·8	...	+ 6·0	...	1	...
3	928 Aug.	17	+ 1·7	...	+ 3·9	...	2	...
4	977 Dec.	12	− 1·4	+ 2·4	+ ·8	+ 2·3	2	1
5	978 June	8	− ·8	+ 4·5	+ 1·3	+ 4·6	4	1
6	979 May	28	+ 1·1	...	+ 3·3	...	2	...
7	985 July	20	+ 7·4	+ 2·2	+ 9·4	+ 2·1	1	2
8	993 Aug.	19	+ 1·0	− 4·1*	+ 3·0	− 4·0*	3	1
9	1004 Jan.	24	+ 2·0	+ 2·9*	+ 3·9	+ 2·8*	1	1
Mean (weighted)			+ 1·01	+ 1·68	+ 3·21	+ 1·65		
Eclipses of the Moon :—								
1	854 Aug.	11	− 0·9	...	+ 1·3	...	1	...
2	856 June	21	− 2·4	...	− 0·2	...	2	...
3	923 June	1	+ 0·5	...	+ 2·8	...	2	...
4	925 April	11	− 2·7	+ 3·1	− 0·5	+ 3·0	4	1
6	929 Jan.	27	+ 6·2	...	+ 8·4	...	1	...
7	933 Nov.	4	− 1·9	...	+ 0·3	...	1	...
9	979 Nov.	6	+ 1·8	+ 2·7	+ 3·9	+ 2·8	2	1
10	980 May	2	+ 0·0	...	+ 2·0	...	1	...
11	981 April	21	− 2·1	− 0·6	+ 0·0	− 0·7	2	2
12	981 Oct.	15	+ 5·2	+ 1·2	+ 7·2	+ 1·3	1	1
13	983 March	1	+ 4·3	+ 6·2	+ 6·3	+ 6·1	3	1
14	986 Dec.	18	+ 7·3	− 1·2	+ 9·3	− 1·3	2	1
15	1002 March	1	− 1·2	...	+ 0·8	..	2	...
Mean (weighted)			+ 1·15	+ 1·34	+ 2·73	+ 1·50		

The mean results being the same for both solar and lunar eclipses, they may be united into one series and yield the values—

Epoch.	Tables B.—Obs.	Tables C.—Obs.	Weight.	Tables B.—Obs.	Tables C.—Obs.	Weight.
849	$\Delta L = -0'.75$	$= +1'.50$	4	ΔE.N.P.D. =		
927	$= +0.11$	$= +2.25$	10	$= +3.10$	$= +3.00$	1
981	$= +1.79$	$= +3.88$	20	$= +2.11$	$= +2.07$	10
999	$= +1.22$	$= +2.44$	6	$= -1.65$	$= -1.65$	2
Mean (weighted)	$= +1.23$	$= +3.21$	36	$= +1.45$	$= +1.41$	13

These Arabian eclipses yield results in accord with the theoretical values of the secular acceleration, and admit of being properly represented by no value of the secular acceleration in mean longitude greater than

$$+6''.20$$

whereas it has been shown that the Ptolemaic eclipses of the Moon can be represented by no value less than

$$+7''.40$$

Hence both series cannot be properly represented by any one value of the secular acceleration, as the mean value

$$+6''.80$$

fails to represent either.

Considering the tabular errors in E.N.P.D., there are seven eclipses by which the error can be determined through the observed duration of the eclipse. These are—

Solar Eclipses.	Tables B.—Obs. E.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$	Lunar Eclipses.	Tables B.—Obs. E.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$
977 Dec. 12	$+2'.4$	$-2'.4$	925 April 11	$+3'.1$	$-3'.1$
978 July 8	$+4.5$	$+4.5$	979 Nov. 6	$+2.7$	$+2.7$
985 July 20	$+2.2$	-2.2	981 April 21	$+0.5$	-0.5
			983 March 1	$+6.2$	-6.2
Mean	$+3.03$	-0.03		$+3.12$	-1.77

The errors regarded as errors in E.N.P.D. are the same from both solar and lunar eclipses, but regarded as due to errors in the adopted longitude of the node, they are discordant. If united, they yield the mean values—

$$\Delta\text{E.N.P.D.} = +3'.09 \qquad \tfrac{1}{11}\Delta\beta = -1'.03$$

Hence, like the Ptolemaic eclipses of the Moon, they indicate a uniform correction to the tabular E.N.P.D. rather than the need of a correction to the tabular longitude of the node.

The six eclipses where the recorded magnitude is available for determining the tabular error in E.N.P.D. are—

Solar Eclipses.		Tables B.—Obs. ΔE.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$	Lunar Eclipses.		Tables B.—Obs. ΔE.N.P.D.	Tables B.—Obs. $\frac{1}{11}\Delta\beta$
985 July	20	$+2'.2$	-2.2	981 April	21	-1.8	$+1.8$
993 Aug.	19	-4.1	-4.1	981 Oct.	15	$+1.2$	$+1.2$
1004 Jan.	24	$+2.9$	$+2.9$	986 Dec.	18	-1.2	$+1.2$
Mean		$+0.33$	-1.13			-0.60	$+1.40$

If these be united they yield the mean values—

$$\Delta\text{E.N.P.D.} = -0'.13 \qquad \tfrac{1}{11}\Delta\beta = +0'.13$$

But the separate results are quite discordant, and disagree with the results obtained from the observed durations. A systematic difference between the results obtained from the magnitude and duration of an eclipse is quite possible in the case of these early observations with unassisted vision, especially in the case of a lunar eclipse, but the observations do not indicate any difference of sensible extent. Thus, in the case of the solar eclipse of 985 July 20, the observed duration and magnitude yield the same value for the error in E.N.P.D. In the case of the lunar eclipse of -173 April 30 the observed duration is $2^{m}.4$ greater than that corresponding to the observed magnitude, whilst in the case of the lunar eclipse of 981 April 21 the observed duration is $4^{m}.5$ less than that indicated by the magnitude.

If the two series be combined they yield the values

	Table B.—Obs.	Table B.—Obs.	Weight.
Solar Eclipses	ΔE.N.P.D $= +1'.68$	$\frac{1}{11}\Delta\beta = -0'.58$	6
Lunar Eclipses	$= +1.53$	$= -0.41$	7
Mean	ΔE.N.P.D. $= +1.60$	$\frac{1}{11}\Delta\beta = -0.49$	13

From the preceding investigations the mean values for the errors in E.N.P.D. for the different epochs may be written as

$-$ 565	ΔE.N.P.D. $= +3'.50$	$\frac{1}{11}\Delta\beta = -3'.84$
$-$ 172	$= +0.85$	$= -1.45$
$+$ 132	$= -0.27$	$= -2.77$
$+$ 980	$= +1.60$	$= -0.49$
$+$1800	$=$ .00	$=$ .00

It is obvious that no correction to the mean motion in E.N.P.D. nor to the mean motion and secular acceleration of the lunar node will serve to represent these values, though it would be possible to bring them into closer accord by means of a term of long period with a coefficient of about 100″, and a period of seven or eight hundred years.

Disregarding the apparently periodical variations, the observed values are best represented by the expression

$$\tfrac{1}{11}\,\Delta\beta = -0''\cdot50\,(T-1800) - 0''\cdot33\,(T-1800)^2$$

but the weight of such a correction must be very small—to be, in fact, a mere possibility.

By assuming that the observed tabular error is due to a gradual change in both E.N.P.D. and argument of latitude β, a closer agreement may be obtained. Separating the observations into those made at the two nodes, the results are

Ascending Node.				*Descending Node.*			
−620	ΔE.N.P.D. =	−0′·60	weight 2	−540	ΔE.N.P.D. =	+5′·14	weight 5
− 2	=	−3·10	2	− 31	=	+1·76	5
+987	=	+1·44	5	+978	=	+1·70	8

These may be regarded as indicating

		Obs.		Obs.	Cal.
−580	ΔE.N.P.D. =	+2′·27	$\frac{1}{11}\Delta\beta$ =	−2′·87	−2′·41
− 17	=	−0·67	=	−2·43	−1·46
+983	=	+1·57	=	−0·13	−0·20

corresponding to the correction

$$\tfrac{1}{11}\,\Delta\beta = +1''\cdot00\,(T-1800) - 0''\cdot30\,(T-1800)^2$$

but it is obvious that the results for the middle epoch are discordant.

It is to be remembered that the observations in the three groups were made under somewhat different conditions. The first group consists of observations made by Babylonian astronomers without the aid of any instrumental means; the second group by Grecian astronomers with some instrumental aid; and the third group by Arabian astronomers by the aid of pinule graduated astrolabes. It is possible, therefore, that the estimates of magnitudes may be systematically different; and it is noteworthy that if it be supposed that the Grecian astronomers under-estimated the magnitude of the five eclipses observed between −200 and +136 by a digit as taken from the southern limbs, it would change the observed errors of the middle group to

$$-\ 2\ \Delta\text{E.N.P.D.} = -1'\cdot26 \qquad -31\ \Delta\text{E.N.P.D.} = +3'\cdot28$$

corresponding to the derived correction

$$-17\ \Delta\text{E.N.P.D.} = +1'\cdot01 \qquad \tfrac{1}{11}\,\beta\Delta = +2'\cdot27$$

thus reducing the discrepancy, but still leaving it far too large for any confidence to be placed in the resulting value for the correction to the node.

How far do the results obtained in the preceding investigations of the observations of these early eclipses agree with Mr Cowell's conclusion that the Moon's argument of latitude shows a secular

acceleration about four and a half seconds greater than that theoretically indicated by the secular decrease in the eccentricity of the terrestrial orbit?

It will be seen that the contact observations of the Ptolemaic eclipses do not indicate the existence of such a correction, and the similar observations of the Arabian eclipses do not extend over a sufficiently long period to enable any conclusion to be formed.

The observed magnitudes of these Ptolemaic eclipses, however, may be regarded as indicating the existence of a secular term of this nature, though of smaller dimensions; but the deduced correction is entitled to little weight, as the separate results are far from accordant unless a considerable term of long period is assumed to exist, and, as they stand, seem more consistent with a secular change in the E.N.P.D. of the Moon.

The magnitude and duration of the early eclipses of the Sun and Moon observed by the Arabian astronomers, likewise, indicate the existence of a correction to the tabular E.N.P.D. of the Moon, which might be ascribed to a secular term in the expression for the Moon's argument of latitude of similar character to that suggested by Mr Cowell; but the separate results show discordances greater than can be ascribed to outstanding errors of observation, which renders it difficult to say whether what is indicated is a change in the adopted secular variation in the argument of latitude, or an analogous change in the E.N.P.D.'s themselves.

When the two series of eclipse observations are united, unfortunately it becomes still less easy to reach a definite conclusion as to which cause the observed deviations from the tabular E.N.P.D. are due to. The observed discordances are so considerable, whatever interpretation be given, that little weight can be assigned to the conclusion which is drawn. Still, though the weight may be small, the observations can be held to support the existence of a secular term in the Moon's argument of latitude greater than can be deduced by theory from the observed decrease in the eccentricity of the terrestrial orbit. Yet they are also consistent, though in a lesser degree, with a secular increase in the observed E.N.P.D., and in this connection it is to be remembered that Tycho Brahe's observations indicate an E.N.P.D. greater by 40″ than that assigned by Hansen's Tables, and that the eclipses of the Sun during the period 1620–1670 indicate an E.N.P.D. from 10″ to 15″ greater than that assigned by the tables.

The only conclusion that can be legitimately drawn from these early eclipse observations appears to be that they are certainly not inconsistent with Mr Cowell's conclusion that the Moon's argument of latitude requires an increased secular acceleration.

In my memoir it was decided that the uncertainty attaching to the correction to the tabular E.N.P.D. indicated by these eclipse observations was too great for them to be taken into account in determining the correction to the tabular value of the longitude of the lunar node.

Natal Observatory:
1906 *August* 17.

On the Early Eclipses. By E. Nevill.

The following notes will serve to elucidate some points in Mr Cowell's paper on the *Ancient Eclipses* in *Monthly Notices*, vol. lxvi. p. 473.

As it was not proposed to use these records of early eclipses of the Sun for deriving corrections to the lunar tables, but only to ascertain how far they were represented by the amended tables, it was judged sufficient to employ approximative forms of calculation; and the data for this purpose were obtained during my visit to England in 1890, from Oppolzer's *Canon der Finsternisse* (Wien, 1887), by applying to his data the corrections necessary to reduce them to the theoretical values of the secular acceleration in mean longitude, longitude of perigee, and longitude of node.

The elements employed by Oppolzer are stated to correspond to the following:—

Mean longitude	ζ = Hansen's Tables +	$\{0''.00 - 26''.34T - 3''.574T^2 - 0.004964T^3\}$
Longitude of Perigee	A = Hansen's Tables +	$\{0.00 - 26.34T - 16.073T^2 - 0.016841T^3\}$
Longitude of Node	B = Hansen's Tables +	$\{-5.50 + 53.06T - 2.956T^2 - 0.004331T^3\}$

where T denotes centuries reckoned from the epoch 1800.

To reduce these to the theoretical values of the secular accelerations, it is necessary to apply the corrections

$$\begin{aligned}&\text{Mean longitude} &&= -\ 2''.496T^2\\ &\text{Longitude of Perigee} &&= +13.128T^2\\ &\text{Longitude of Node} &&= +\ 2.672T^2\end{aligned}$$

At the same time, in order to bring the amended tables into accord with modern observations, it is necessary to apply the further corrections

$$\begin{aligned}&\text{Mean longitude} &&= -30''.00T\\ &\text{Longitude of Perigee} &&= +10.80T\\ &\text{Longitude of Node} &&= +\ 5.00T\end{aligned}$$

Hence the total corrections to be applied to the elements of the Moon's orbit employed by Oppolzer in order to reduce them to the theoretical values are:

$$\begin{aligned}\Delta\zeta &= +0''.00 - 3''.66T - 2''.496T^2\\ \Delta A &= +0.00 + 37.14T + 13.128T^2\\ \Delta B &= +0.00 - 48.06T + 2.672T^2\end{aligned}$$

Oppolzer's formulæ for calculating the latitude ϕ and longitude λ of the position of the curve of central totality for the beginning, middle, and end of an eclipse are

Beginning	$\sin\phi$	$= -\cos\delta'\cos(N'+W)$	$\lambda = -\mu + \frac{15^\circ}{n}\cos W - \tan(N'+W)\operatorname{cosec}\delta'$
Middle	$\sin(\phi-\delta') =$	$\sin W \operatorname{cosec} N'$	$\lambda = -\mu - \frac{15^\circ}{n}\cot N' \sin W$
End	$\sin\phi$	$= +\cos\delta'\cos(N'-W)$	$\lambda = -\mu - \frac{15^\circ}{n}\cos W - \tan(N'-W)\operatorname{cosec}\delta'$

where

$$\sin W = \frac{\sin(b-b')}{\sin(\pi-\pi')} \cdot \sin(N'+h)$$

$$\tan h = \tan \epsilon \cdot \cos L'$$

$$\mu = 15^\circ.H - \frac{15^\circ}{n} \cdot \sin W \cdot \cot(N'+h) - \text{Equation of Time.}$$

Here

H denotes the epoch of conjunction measured from Greenwich Noon.
b, b' denote the latitude of the Moon and Sun.
π, π' denote the horizontal parallaxes of the Moon and Sun.
L', δ' denote the true longitude and declination of the Sun.
ϵ denotes the obliquity of the Ecliptic.

The other quantities are taken as they stand from Oppolzer's *Canon.*

It is obvious that it is only W and μ that will suffer any material change by the introduction of the correction $\Delta\zeta$, ΔA, and ΔB.

Then α denoting the Moon's mean anomaly and β the Moon's argument of latitude,

$$\Delta \sin W = \Delta B \cdot \cos(b-b') \operatorname{cosec}(\pi-\pi') \sin(N'+h)$$

where approximately

$$\Delta B = -\cos\beta\{(0{\cdot}092 - 0{\cdot}006 \cos\alpha)\Delta B + 0{\cdot}007\Delta\zeta\}$$

and

$$\Delta\mu = 15^\circ\{\Delta H - \Delta B \cdot n^{-1} \cdot \cos(N'+h)\}$$

where approximately

$$\Delta\mu = -(28^\circ{\cdot}65 - 1{\cdot}38 \cos\alpha)\Delta\zeta + 2^\circ{\cdot}64 \cos\alpha \cdot \Delta A + 0^\circ{\cdot}25\Delta B$$

The changed path of the curve of central totality can be deduced from the comparison of the new positions of the beginning, middle, and the end of eclipse with those given by Oppolzer.

For the required purpose this approximation was sufficiently accurate.

It was in the same manner that the approximate paths of the different eclipses were calculated on Mr Cowell's original data. In the case of the eclipse of −1116 June 18 an accidental error in copying out the final correction as plus instead of minus led to an inaccurate result, and this eclipse cannot have been that observed at Babylon.

I cannot concur with Mr Cowell's method of bringing −1062 July 31 within the month of *Sivan.* The intercalary months were never inserted at the beginning of the year, and the first of *Nisan* could not have been later than the early part of April, so that the month of *Sivan* must have ended in the beginning of July at the latest. Nor can I concur with Mr Maunder in supposing that though July 31st could not have fallen within *Sivan* according to the known Babylonian calendar, yet that it may be legitimately assumed that, as the epoch is prior to the era of *Nabonasser*, the calendar may be supposed to be quite different from that known to

be used after that era. If such arbitrary assumptions have to be made, the records of these eclipses are absolutely valueless.

On this question I would observe, merely, that it must not be forgotten that for many centuries prior to the epoch of this eclipse the Babylonians had possessed a well-regulated definite calendar, sufficiently certain and predicate to enable its being incorporated in deeds, leases, and commercial agreements. For centuries the beginning of the year was fixed by the occurrence of the equinox, and it is most unlikely that they were dependent for the fixing of this event on the observation of actual New Moon, or on the heliacal or other fancy rising or setting of a star, when, in the passage of the shadow from the Sun over a fixed line on the Temple floor or courtyard, the line midway between the longest and shortest shadow during the year, they had a means of fixing the date of the equinox to the very day. Then, when the equinox fell after the first half of the fixed month, the year was made full by the insertion of an intercalary month at the end of the year. The reckoning of time in this manner by the dimensions of the shadows occurs in the early history of all races, and is universal in the East even now. The heliacal rising and setting of stars and the visibility of the new moon would have their religious and astrological signification, but so clumsy and indirect a method of fixing the equinox would not have been used when so much easier a method was available. Recently, every year has brought further evidence, in the shape of astronomical records and calculations, that the Assyrians, Babylonians, and even the early Chaldeans possessed a much better knowledge of astronomy and much better means of making astronomical observations than had been assigned to them, and that they could measure epochs and intervals of time with some certainty, probably by the equivalent to a dial on the floor of the Temple or its courtyard, as well as by contrivances of the nature of clepsammia or clepsydra. This, however, is beyond the present subject.

Fuller discussion of the eclipse seen at Babylon "on the 26th *Sivan*" is useless until the full details of the record are before us, and it is possible to critically examine the reasons for assigning it to some date in the eleventh century before our era, for at present there exist no data for estimating its value as the record of an eclipse at any place or at any epoch. If the record can be established as proving the occurrence of a total eclipse visible at Babylon in the eleventh century before our era, it is undoubtedly of the very highest importance as fixing both astronomical and chronological elements, but we must have the record with all ancillary evidence before its true bearings can be properly discussed.

Mr Cowell points out that the failure of the observations of the Sun during the period 1750–1900 to show any signs of the assumed secular acceleration in its mean longitude may be due to the existence of an unknown term of very long period. That is indubitable; and on that basis it must be admitted that these

observations of the Sun cannot be said to be inconsistent with the extended hypothesis.

The evidence that has been advanced serves to show that the facts brought forward by Mr Cowell in support of his views can be explained without having recourse to a new secular acceleration in the mean motion of the Earth, so that these facts do not suffice to establish the existence of such a secular acceleration. But if this is all, they certainly do not suffice to disprove the existence of such a term. If that is to be done, it must be by further direct evidence, to which I have no access, though it may exist. During past years in my researches I have repeatedly found evidence of an unexplained apparent secular acceleration in the motion of the Moon's argument of latitude, but I have doubted its reality, as no explanation of its origin seemed available. The explanation advanced by Mr Cowell, that it might be due to a secular acceleration in the mean motion of the Earth, did not occur to me, or I should have taken the necessary steps to obtain the data to investigate the matter. This, I presume, will be done by Mr Cowell. I do not see any theoretical explanation of such a secular acceleration which does not seem to involve great difficulties. Mr Cowell's tentative explanation (*Monthly Notices*, vol. lxvi. p. 352) I think he will find untenable on review.

Natal Observatory:
1906 *August* 23.

The Mediæval Eclipses of Celoria. By P. H. Cowell.

Professor Celoria has collected a number of references to the total eclipses of 1239 June 3 and 1241 October 6. He considers that the former eclipse was certainly total at Piacenza and at Lesina, and that the latter was total at Stadt and at Ellwangen. He draws the limits of totality according to Hansen's Tables (without Newcomb's corrections), and concludes that these tables are in serious error at the epoch of the two eclipses.

Professor Celoria's diagrams show that the conditions that he imposes are very stringent ones. The zone of totality for the 1239 eclipse is to be displaced not less than enough to secure totality at Piacenza, and not so much as to destroy totality at Lesina, and these limits are very narrow. They become still narrower if it be remembered that Hansen's semi-diameter is considerably too large for eclipse purposes. A similar remark applies to the eclipse of 1241. Professor Celoria's assumptions, therefore, lead to two equations of condition, one for each eclipse, in which the margin of uncertainty is very slight. The combination of the two should therefore give an excellent determination of the corrections required by the mean elongation and by the argument of latitude.

The form of the equations of condition can be most simply found. Let k denote, as in my previous papers on eclipses, the